1. For the RLC circuit shown below, determine the following:

   a. Total impedance “seen” by the source as a function of source frequency, $\omega$.
   b. Source frequency (in Hertz) when the total impedance magnitude is a minimum and thereby current a maximum.
   c. At the frequency of part b, peak value of $I$.
   d. At the frequency of part b, voltage across $R$ in Volts rms.
   e. Quality factor $Q_s$ of the circuit. Note: Inductor ideal.

2. Given a resonant frequency of 1,000 rad/sec and a circuit quality factor $Q_s = 10$, determine $L$ and $C$ in the circuit below. Note: Inductor ideal.
3. For the parallel RLC circuit shown below, determine the following:

a. Total admittance “seen” by the source as a function of source frequency, \( \omega \).

b. Source frequency (in Hz) when the total impedance magnitude is a maximum and thereby output voltage \( V_o \) a maximum.

c. At the frequency of part b, peak value of \( V_o \).

d. At the frequency of part b, current through R in A rms.

e. The circuit’s quality factor \( Q_p \). Note: Inductor ideal.

4. For the parallel RLC circuit shown below:

a. Determine the frequency (in Hz) for which \( V_o \) is a maximum.

b. With an 80\( \Omega \) load connected to terminals ab and at a source frequency of part a, what value of R is required to deliver maximum power to the 80\( \Omega \) load?

c. Under the conditions of part b, what is the amount of power (in mW) delivered to the load?
Objective
To observe the effects of resonance in both series and parallel RLC circuits.

Workbench Equipment
- Function Generator, Agilent 33120A
- Digital Multimeter, Agilent 34401A
- Digital Multimeter, Fluke 8840A
- Resistor Box IV, 100Ω/250Ω/500Ω
- Resistor Decade Box, 10KΩ step
- Inductor Decade Box, 10mH step: 0 – 100mH
- Capacitor Decade Box, 0.1μF step: 0 – 1.1μF
- Capacitor Decade Box, 1μF step: 0 – 10μF
- RLC Bridge, Gen Rad 1659

Check-out Equipment, 20-111 window
- Banana to banana, 3 pairs, red/black
- BNC to banana, quantity 1
- Short leads, quantity 6, 1 bag

Background
The resonance phenomenon is important in electrical circuits because it enables selection of a desired signal frequency from a range of frequencies, as in radio and TV receivers. Resonance is not always a desirable feature of a system since it can also cause undesired power dissipation leading to possible system degradation or failure. In resonant electric circuits, there is an interchange of energy between capacitive and inductive elements. There is also a loss factor associated primarily with the effective resistance of practical (non-ideal) inductors and capacitors.

Series Resonance
Fig. 7-1 illustrates a series RLC circuit. The total impedance “seen” by the source (Zeq) in this circuit is:

$$Zeq = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right) = R \pm jX_{eq}$$  \hspace{1cm} (7-1)

At the resonant frequency ω₀ (ω₀ = 2πf₀), Xeq = 0 since \(\omega L = \frac{1}{\omega C}\) when \(\omega = \omega_0\).
\[ X_{eq} = \omega_o L - \frac{1}{\omega_o C} = 0 \Rightarrow \omega_o = \frac{1}{\sqrt{LC}} \quad \text{or} \quad f_o = \frac{1}{2\pi\sqrt{LC}} \] (7-2)

At this frequency, \( Z_{eq} \) is purely real (resistive). Since \( |Z_{eq}| = \sqrt{R^2 + X_{eq}^2} \), \( |Z_{eq}| \) reaches a minimum value at resonance.

\[ |Z_{eq}|_{\omega_o} = R \quad \text{and} \quad I_{\omega_o} = \frac{V}{Z_{eq}} \rightarrow \frac{V}{R} \] (7-3)

Since \( I = V / Z_{eq} \), the current reaches a maximum value when \( Z_{eq} = R \).

At frequencies (\( \omega_2, \omega_1 \)) above and below the resonant frequency where \( X_{eq} = \pm R \).

\[ |Z_{eq}|_{\omega_1,\omega_2} = R\sqrt{2} \quad \text{and} \quad \left| I_{\omega_1,\omega_2} \right| = \frac{V}{R\sqrt{2}} \] (7-4)

These two frequencies are referred to as the half-power frequencies or cut-off frequencies.

Solving for frequency \( \omega_2 \): \( X_{eq} = +R \)

\[ \omega_2 L - \frac{1}{\omega_2 C} = +R \Rightarrow \omega_2^2 - \frac{R}{L} \omega_2 - \frac{1}{LC} = 0 \] (7-5)

\[ \omega_2 = \frac{R}{2L} \pm \sqrt{\left( \frac{R}{2L} \right)^2 + \frac{1}{LC}} \Rightarrow \omega_2 = \frac{R}{2L} + \sqrt{\left( \frac{R}{2L} \right)^2 + \frac{1}{LC}} \] (7-6)

Note that the positive sign must be selected to avoid negative frequency values.

Solving for frequency \( \omega_1 \): \( X_{eq} = -R \)

\[ \omega_1 L - \frac{1}{\omega_1 C} = -R \Rightarrow \omega_1^2 + \frac{R}{L} \omega_1 - \frac{1}{LC} = 0 \] (7-7)

\[ \omega_1 = -\frac{R}{2L} \pm \sqrt{\left( \frac{R}{2L} \right)^2 + \frac{1}{LC}} \Rightarrow \omega_1 = -\frac{R}{2L} + \sqrt{\left( \frac{R}{2L} \right)^2 + \frac{1}{LC}} \] (7-8)

Again the positive sign must be specified to avoid negative frequencies.

The circuit bandwidth \( BW \) is defined as the frequency range over which the current is \( \sqrt{2} \) times maximum value or greater. Using equations (7-6) and (7-8):

\[ BW = \omega_2 - \omega_1 = \frac{R}{L} \] (7-9)

The quality factor \( Q_s \) is defined as the ratio of the resonant frequency to the circuit bandwidth:

\[ Q_s = \frac{\omega_o}{BW} = \frac{L}{R\sqrt{LC}} = \frac{1}{\frac{R}{\sqrt{LC}}} \] (7-10)

**Parallel Resonance**

Fig. 7-2 illustrates a parallel RLC circuit. The total admittance \( Y_{eq} \) of this circuit is

\[ Y_{eq} = G + j\omega C + \frac{1}{j\omega L} = G + j\left( \omega C - \frac{1}{\omega L} \right) = G \pm jB_{eq} \] (7-11)

where \( G \) (conductance) is the reciprocal of resistance and \( B \) (susceptance) is the reciprocal of reactance.

\[ G = \frac{1}{R} \quad \text{and} \quad B = \frac{1}{X} \quad \text{therefore} \quad B_C = \frac{1}{X_C} = \omega C \quad \text{and} \quad B_L = \frac{1}{X_I} = \frac{1}{\omega L} \] (7-12)
At the resonant frequency $\omega_0 (\omega_0 = 2\pi f_0)$, $B_{eq} = 0$ since $\omega C = \frac{1}{\omega L}$ when $\omega = \omega_0$.

$$B_{eq} = \omega_0 C - \frac{1}{\omega_0 L} = 0 \quad \Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad \text{or} \quad f_0 = \frac{1}{2\pi\sqrt{LC}} \quad (7-13)$$

The resonant frequency equation is the same for both series and parallel RLC circuits. At this frequency, $Y_{eq}$ is purely real (conductive). Since $G_Y = \sqrt{G^2 + B_{eq}^2}$, $|Y_{eq}|$ reaches a minimum value at resonance.

$$|Y_{eq}|_{\omega_0} = G \quad \text{and} \quad V|_{\omega_0} = \left| \frac{I}{Y_{eq}|_{\omega_0}} \right| = \frac{I}{G} \quad (7-14)$$

Since $V = I / Y_{eq}$, the voltage reaches a maximum value when $Y_{eq} = G$.

At frequencies ($\omega_2$, $\omega_1$) above and below the resonant frequency where $B_{eq} = \pm G$,

$$|Y_{eq}|_{\omega_1,\omega_2} = G\sqrt{2} \quad \text{and} \quad V|_{\omega_1,\omega_2} = \left| \frac{I}{Y_{eq}|_{\omega_1,\omega_2}} \right| = \frac{I}{G\sqrt{2}} = \frac{V}{\sqrt{2}} \quad (7-15)$$

These two frequencies are referred to as the half-power frequencies or cut-off frequencies.

Solving for frequency $\omega_2$: $B_{eq} = +G$

$$\omega_2 C - \frac{1}{\omega_2 L} = +G = \frac{1}{R} \Rightarrow \omega_2^2 = \frac{1}{RC} \omega_2 - \frac{1}{LC} = 0 \quad (7-16)$$

$$\omega_2 = \frac{1}{2RC} \pm \sqrt{\left( \frac{1}{2RC} \right)^2 + \frac{1}{LC}} \Rightarrow \frac{1}{2RC} \pm \sqrt{\left( \frac{1}{2RC} \right)^2 + \frac{1}{LC}} \quad (7-17)$$

Note that the positive sign must be specified to avoid negative frequency values.

Solving for frequency $\omega_1$: $B_{eq} = -G$

$$\omega_1 C - \frac{1}{\omega_1 L} = -G = -\frac{1}{R} \Rightarrow \omega_1^2 + \frac{1}{RC} \omega_1 - \frac{1}{LC} = 0 \quad (7-18)$$

$$\omega_1 = -\frac{1}{2RC} \pm \sqrt{\left( \frac{1}{2RC} \right)^2 + \frac{1}{LC}} \Rightarrow -\frac{1}{2RC} \pm \sqrt{\left( \frac{1}{2RC} \right)^2 + \frac{1}{LC}} \quad (7-19)$$

Again the positive sign must be specified to avoid negative frequencies. The circuit bandwidth $BW$ is defined as the frequency range over which the voltage is $\sqrt{2}$ times maximum value or greater. Using equations (7-17) and (7-19):

$$BW = \omega_2 - \omega_1 = \frac{1}{RC} \quad (7-20)$$

The quality factor $Q_p$ is defined as the ratio of the resonant frequency to the circuit bandwidth:

$$Q_p = \frac{\omega_0}{BW} = \frac{RC}{\sqrt{LC}} = R \frac{C}{L} = \frac{1}{Q_l} \quad (7-21)$$
Procedure 1 Series Resonance (Determining Resonant Frequency)

- Use an impedance bridge (set to 1 KHz) to measure and record L and C values specified in Table 7-1. Also, record measured value of R (100Ω) in Table 7-1.
- From measured L and C values, calculate and record resonant frequencies for each LC combination in Table 7-1.
- Construct circuit of Fig. 7-3; for each LC combination: (connect 50mH / 0.5µF combo last)
- Set the function generator (FG) to high Z output – see Lab 6 procedure 1 and adjust settings to produce a sine wave with the calculated resonant frequency and amplitude to yield a |VC| of approximately 1Vrms. Record loaded |VS| level (Vrms) in Table 7-2.
- Maintain the FG amplitude setting and adjust the frequency only to maximize |VR|. Record this frequency in Table 7-1 and the maximum |VR| value (Vrms) in Table 7-2.
- Calculate the percent difference between calculated and measured resonant frequencies for each LC combination and record in Table 7-1.

![Fig. 7-3 Circuit for Determining Series Resonant Frequency](image)

### Table 7-1 Measured and Calculated Resonant Frequencies

<table>
<thead>
<tr>
<th></th>
<th>Nominal L = 30mH</th>
<th>Nominal L = 50mH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured L</td>
<td>_______ mH</td>
<td>_______ mH</td>
</tr>
<tr>
<td>Nominal C = 0.5µF</td>
<td>f₀ = __________</td>
<td>f₀ = __________</td>
</tr>
<tr>
<td>Measured C</td>
<td>_______ µF</td>
<td>_______ µF</td>
</tr>
<tr>
<td>Nominal C = 2.0µF</td>
<td>f₀ = __________</td>
<td>f₀ = __________</td>
</tr>
<tr>
<td>Measured C</td>
<td>_______ µF</td>
<td>_______ µF</td>
</tr>
<tr>
<td>Measured R</td>
<td>_______ Ω</td>
<td>_______ Ω</td>
</tr>
</tbody>
</table>

### Table 7-2 Procedure 1 Voltage Levels

<table>
<thead>
<tr>
<th></th>
<th>Nominal L = 30mH</th>
<th>Nominal L = 50mH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal C = 0.5µF</td>
<td>Vₛ (Vrms)₁</td>
<td>Vₛ (Vrms)₁</td>
</tr>
<tr>
<td>Nominal C = 2.0µF</td>
<td>Vₛ (Vrms)₂</td>
<td>Vₛ (Vrms)₂</td>
</tr>
<tr>
<td>Vₚₚₐₓₚ (Vrms)₂</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Footnotes: 1) Amplitude adjustment to |VC| = 1Vrms  2) Frequency adjustment to max |VR|
Procedure 2: Series Resonant Circuit Parameters

- If you connected 50mH / 0.5µF LC combo last, your circuit is now the circuit of Fig. 7-4.
- For frequencies in the range of 600Hz to 1700Hz (100Hz increments): see Table 7-3
  - Measure and record the rms voltage across the 100Ω resistor.
  - Take additional measurements in smaller (1Hz) increments near resonant frequency.
  - Include these additional measurements in Excel plot data table, but not in Table 7-3.
- Plot voltage as a function of frequency. From your plot:
  - Determine the half-power frequencies (f₁ and f₂) and calculate bandwidth (BW).
  - Determine the resonant frequency f₀.
- Calculate f₁, f₂ using measured values of R, L and C. Calculate BW, f₀ and Qₛ.
  - To compute f₁ and f₂ use total series resistance = measured R + inductor’s DC resistance (measure with ohmmeter) + 50Ω (internal resistance of FG). Record inductor’s DC resistance in Table 7-3.
- Record measured Qₛ (from plot) and calculated Qₛ along with % percent error in Table 7-3.

![Fig. 7-4 Series RLC Circuit](image)

<table>
<thead>
<tr>
<th>f (Hz)</th>
<th>Vₛ (mV)</th>
<th>f (Hz)</th>
<th>Vₛ (mV)</th>
<th>f (Hz)</th>
<th>Vₛ (mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>1000</td>
<td>1400</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>700</td>
<td>1100</td>
<td>1500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>1200</td>
<td>1600</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>900</td>
<td>1300</td>
<td>1700</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7-3 Series RLC Frequency Response Data

Procedure 3 Parallel Resonant Circuit Parameters

- Use impedance bridge (set to 1 KHz) to measure L and C values of Fig. 7-5. Also, measure both resistors 10KΩ and 50KΩ with ohmmeter. Record in Table 7-4.
- Use measured values to calculate resonant frequency for LC combo, record in Table 7-4.
- Construct the circuit of Fig. 7-5.
- Set the FG to high Z output – see Lab 6 procedure 1 and adjust settings to produce a sine wave with the calculated resonant frequency and a maximum amplitude of 16Vpp. Record the resulting V₁ (Vrms) in Table 7-4.
- For frequencies in the range of 1000Hz to 1275Hz (25Hz increments): see Table 7-4
  - Maintain V₁ at the value recorded in the previous step by adjusting FG amplitude. This will simulate a constant current source.
  - Measure and record V₂ in rms voltage in Table 7-4
  - Take additional measurements in smaller (1Hz) increments near resonant frequency.
  - Include these additional measurements in Excel plot data table, but not in Table 7-4.
• Plot voltage as a function of frequency. From your plot:
  o Determine the half-power frequencies \( f_1 \) and \( f_2 \) and calculate BW.
  o Determine the resonant frequency \( f_0 \). Record in Table 7-4. Calculate % Diff.
• Calculate \( f_1 \), \( f_2 \) using measured values of \( R \), \( L \) and \( C \). Calculate BW, \( f_0 \) and \( Q_p \).
  o To compute \( f_1 \) and \( f_2 \) use the total resistance of the parallel combination (“seen” from the load) of the 50K\( \Omega \) and 10 K\( \Omega \). The internal resistance (50\( \Omega \)) of the FG can be ignored since it is << 50K\( \Omega \).
• Record measured \( Q_p \) (from plot) and calculated \( Q_p \) along with % percent error in Table 7-4.

![Fig. 7-5 Parallel RLC Circuit](image)

<table>
<thead>
<tr>
<th>Nominal L = 100mH</th>
<th>Measured L = ________ mH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal C = 0.2( \mu )F</td>
<td>( f_0 = )</td>
</tr>
<tr>
<td>Measured C = ________ ( \mu )F</td>
<td></td>
</tr>
<tr>
<td>Measured 10K = ________ ( \Omega )</td>
<td></td>
</tr>
<tr>
<td>Measured 50K = ________ ( \Omega )</td>
<td></td>
</tr>
<tr>
<td>( f ) (Hz)</td>
<td>( V_2 ) (mV)</td>
</tr>
<tr>
<td>1000</td>
<td>1100</td>
</tr>
<tr>
<td>1025</td>
<td>1125</td>
</tr>
<tr>
<td>1050</td>
<td>1150</td>
</tr>
<tr>
<td>1075</td>
<td>1175</td>
</tr>
</tbody>
</table>

Table 7-4 Parallel RLC Frequency Response Data

Discussion

1. For both circuits, series RLC and parallel RLC, comment on any differences between theoretical and experimental values of \( Q \), \( f_0 \), \( f_2 \) and \( f_1 \).
2. Compare the two curves obtained for the series RLC and the parallel RLC circuits. Comment on what effect \( Q \) has on the curve shape and on bandwidth.
3. What application would a high \( Q \) circuit be desirable? Can you think of an application of a low \( Q \) circuit? Hint: Low \( Q \) circuits are referred to as broadband.