IMPORTANT – SHOW ALL WORK!
1. Show that resistance multiplied by capacitance has units of time.

2. For the circuit shown, at $t=0$ the switch is closed. $R = 40\, \Omega$, $C = 0.02\, \mu F$, $V_s = 4\, \text{Volts}$
   Note: $V_O(0) = 0\, \text{V}$ (initially uncharged)
   a. Sketch $V_c(t)$ for $0 \leq t \leq 5\tau$. Label y axis of plot with $V_c$ values at $t = \tau$, $2\tau$, $3\tau$, $4\tau$ and $5\tau$.
   b. How long (ms) is required for the capacitor to reach steady-state?
   c. What is the capacitor current at $t = \tau$, $t = 3\tau$, $t = \text{steady-state}$.

3. An initially charged $0.02\, \mu F$ capacitor, $V_C(0) = 4\, \text{V}$, is connected across a $40\, \Omega$ resistor.
   a. At what time will the voltage across the capacitor equal $2\, \text{V}$?
   b. At what time will the voltage across the capacitor equal $0.5\, \text{V}$?
4. Inductors and capacitors; circle correct answer:
   a. At steady-state, an inductor behaves like a/an (short / open) circuit relative to DC current.
   b. At steady-state, a capacitor behaves like a/an (short / open) circuit relative to DC current.

5. Calculate I and V for the following circuit at steady-state: both resistors in ohms.
Objective

To investigate DC transient response of a RC circuit and to analyze a RLC circuit at DC steady-state.

Workbench Equipment

- Function Generator, Agilent 33120A
- DC Power Supply, Agilent E3640A
- Digital Multimeter, Agilent 34401A
- Digital Oscilloscope, Agilent 54621A
- Resistor Box II, 10Ω/25Ω/40Ω/130Ω/269Ω/562Ω
- Resistor Decade Boxes, 10 kΩ step
- Inductor Decade Box, 10mH step: 0 – 100mH
- Capacitor Decade Box, 0.01µF step: 0 – 1.1µF
- Impedance Bridge, Gen Rad 1659

Check-out Equipment, 20-111 window

- Scope Probe (10:1), 2
- BNC to banana lead
- Banana to banana, 3 pairs, red/black
- Short leads, quantity 6, 1 bag

Background

Transient Response of RC Circuits

Capacitor Discharging: When an initially charged capacitor is connected to a resistance as shown in Fig. 5-1, as time progresses, the capacitor’s stored energy is dissipated as heat by the resistor. As stored energy is released, the voltage across the capacitor plates, as well as charge on the plates ($V = Cq$) diminishes to zero at steady state. When plate voltage equals zero, there is no electric field within the dielectric between the plates and stored energy is completely expended.

\[
\text{KCL at top node of Fig. 5-1 with switch closed (t > 0)} \quad i_C = i_R
\]

\[
\text{Recall: } i_C = -C \frac{dV_C(t)}{dt} \quad \text{(passive sign convention)} \quad \frac{dV_C(t)}{dt} + \frac{V_C(t)}{R} = 0 \quad (5-1)
\]

Current exits the higher voltage plate.

Fig. 5-1 RC Circuit (Capacitor Discharging)
{1st-order homogeneous DEQ}  \[
\frac{dV_C(t)}{dt} + \frac{V_C(t)}{RC} = 0
\]  (5-2)

{General solution}  \[
V_C(t) = Ae^{st}
\]  (5-3)

Sub general solution into 1st-order homogeneous DEQ
\[
sAe^{st} + \frac{1}{RC} Ae^{st} = 0
\]  (5-4)

Solving for s:
\[
(s + \frac{1}{RC}) Ae^{st} = 0 \Rightarrow (s + \frac{1}{RC}) = 0 \Rightarrow s = -\frac{1}{RC}
\]  (5-5)

\[
V_C(t) = Ae^{-t/RC}
\]  (5-6)

Solve for A at t = 0:
\[
V_C(0) = Ae^{0/RC} = A = V_0
\]  (5-7)

Natural Response (Due to Stored Energy Only):  \[
V_C(t) = V_0e^{-t/\tau} \text{ where } \tau = RC
\]  (5-8)

Capacitor voltage (5-8) decays exponentially as time advances. After five time constants (t = 5RC), the capacitor for all practical purposes has reached the steady-state voltage of zero (e^{-5} = 0.00674) and the capacitor is considered to be depleted of stored energy.

Capacitor Charging: When a source is applied to an RC circuit as shown in Fig. 5-2, \(V_C < V_S\), the electrical charge on the capacitor plates increases as time progresses. Therefore, voltage across the capacitor plates \(V = Cq\) increases as well as charge. This increase in capacitor voltage produces a stronger (more intense) electric field within the dielectric between the plates and the capacitor stores increasing energy \(W_C = \frac{1}{2}CV^2\) as time advances until steady-state is achieved.

![Fig. 5-2 RC Circuit (Capacitor Charging)](image)

Capacitor initially charged:  \(V_C(0) = V_o\) where \(V_o < V_S\)

Complete Response = Natural Response + Forced Response
(due to stored energy) (due to independent source)

Natural Response:  \(V_C(t) = V_0e^{-t/RC}\)  {Derived Previously (5-8)}

KCL at top node of Fig. 5-2 with switch closed (t > 0)  \[i_C = i_R\]

Recall:  \(i_C = C \frac{dV_C(t)}{dt}\) (passive sign convention)  \[
\frac{V_S - V_C}{R} - C \frac{dV_C}{dt} = 0
\]  (5-9)

Current enters higher voltage plate.
{\(V_S\) is a constant, DC source} \[ \frac{dV_c}{dt} + \frac{V_c}{RC} = \frac{V_s}{RC} \] (5-10)

{Forced solution also a constant} \[ V_c(t) = X_F \] (5-11)

Sub in \(X_F\):
\[ 0+ \frac{X_F}{RC} = \frac{V_s}{RC} \quad \Rightarrow \quad X_F = V_S \] (5-12)

Complete Response:
\[ V_c(t) = Ae^{-t/RC} + V_S \] (5-13)

Solve for \(A\) at \(t = 0\):
\[ V_C(0) = V_o = Ae^{-0/RC} + V_S \quad \Rightarrow \quad A = V_o - V_S \] (5-14)
\[ V_C(t) = V_S + (V_o - V_S)e^{-t/RC} \] (5-15)
\[ V_C(t) = V_o e^{-t/RC} + V_S(1 - e^{-t/RC}) \] (5-16)

Complete Response = Natural Response + Forced Response 
(due to stored energy) (due to independent source)

For initially uncharged Capacitor: \(V_o = 0\) \[ V_C(t) = V_S(1 - e^{-t/RC}) \] (5-17)

As can be seen from (5-17), for all practical purposes, capacitor voltage increases from zero volts to a steady-state value of \(V_s\) after five time constants \((1 - e^{-5} = 0.99326)\) and the capacitor is considered to be fully charged.

**Capacitors & Inductors Under DC Steady-State Conditions**

Inductors and capacitors have the following voltage – current relations:

\[ V_L = L \frac{dI(t)}{dt} \] (5-18)
\[ I_C = C \frac{dV(t)}{dt} \] (5-19)

For DC circuits, all time derivatives are zero since all currents and voltages are constant. Therefore, under DC conditions:
\[ V_L = 0 \text{ for any current (same V-I relation as zero ohms = short)} \] (5-20)
\[ I_C = 0 \text{ for any voltage (same V-I relation as infinite ohms = open)} \] (5-21)

Hence, for DC conditions, inductors “act” as short-circuits and capacitors “act” as open-circuits.

**Oscilloscope Basics**

To view the transient response of an RC circuit, an oscilloscope (“scope”) is used. A scope converts an electrical signal into a visible trace on a display screen. The display screen is a graph of voltage (vertical axis) versus time (horizontal axis). Vertical grid lines (or divisions) on the display screen have numerical values relative to the volts per division setting; horizontal screen divisions have numerical values relative to the time per division setting. Both settings can be manually adjusted or the Auto-Scale button can be pressed and the scope will automatically set the volts per division setting and time per division setting for optimum signal visibility, see Fig. 5-3 for an example of a displayed sinusoidal voltage.
A common scope measurement method is to use vertical cursors (dashed lines) to measure signal amplitude such as peak or peak-to-peak or to use horizontal cursors (dashed lines) to measure the time difference between two points on a signal such as measuring period, see Fig. 5-4.

In addition, the Agilent 54621A oscilloscopes have a feature, Quick Measure (button to the right of the Cursors button), that automatically displays amplitude, period, and/or frequency, as well as many other signal parameters without the use of cursors.

Most often, a scope probe is used to transfer the signal under test to the scope input. The most popular scope probe (and used in this lab) is the 10:1 scope probe. The ratio 10:1 refers to the factor of attenuation (loss) between the scope probe tip and the scope input. For example, if the scope probe is connected to a 10 V<sub>peak</sub> signal, the signal amplitude at the scope input is 1 V<sub>peak</sub>. In other words, the scope probe has attenuated (decreased amplitude) by a factor of ten. The scope has a probe adjust setting which compensates for this loss in amplitude; the displayed amplitude matches the signal amplitude at the scope tip.

A BNC to BNC lead or BNC to banana lead can also be used to connect a signal to a scope input. These leads have minimal attenuation and are considered ideally to be shorts. The main advantage a scope probe has over a lead is higher input impedance which means less loading. Recall from experiment one; an ideal voltmeter has infinite internal resistance. Scopes can be thought of as sophisticated voltmeters and likewise have ideal infinite internal resistance / impedance.
Scopes commonly have multiple inputs (also called channels) allowing multiple simultaneous signals to be displayed. Our lab scopes are dual channel (two inputs) and in this experiment one input will be used as a “triggering” source or signal.

Scope triggering can be thought of as synchronized picture taking. A signal displayed (“picture”) on a scope screen is the result of many consecutive digitized samples. This process of picture taking (i.e., producing signal on scope screen) must be synchronized to a unique point on a waveform that repeats, otherwise the displayed signal becomes unstable. The triggering source provides the unique point, most often the triggering signal has a sharp transition, such as a square wave does, and either the high to low transition (falling-edge or trailing-edge) or the low to high transition (rising-edge or leading-edge) is used as the unique point for synchronization.

Procedure 1: RC Transient Response

- Use impedance bridge set at 100 Hz to measure capacitor value and use ohmmeter to measure resistor value used to construct circuit of Fig. 5-5 and record in Table 5-1.
- Set function generator (FG) to high-Z output: shift-menu, right arrow to the SYS menu, down arrow to OUTPUT TERM menu, down arrow to ‘50 ohms’, right arrow to select Hi Z; enter.
- Set function generator to a square wave output with a minimum value of 0 V and a maximum value of 4 V (amplitude = 4Vpp with DC offset = 2V). Set frequency of square wave to 100Hz and use a duty cycle of 50%.
  - Duty cycle refers to the amount of time a square wave is equal to its largest amplitude (4V in this case) compared to its period.
- Connect channel 1 scope probe between node A and ground (black clip to ground).
- Connect channel 2 scope probe between node B and ground (black clip to ground).
  - Verify scope probe setting is 10:1. Press channel button, channel menu appears at bottom of scope screen and then press Probe button and adjust with knob located between time per division setting and channel 1 volt per division setting.
- Set both scope channels to DC coupling (on channel menu) and set both channel vertical sensitivity to 1V/div.
- Set horizontal sensitivity to 1millisec/division.
- Set trigger settings as follows:
  - Press Mode Coupling button and select auto level.
  - Press Edge button and choose Channel 1 and rising edge.
  - Note: May need to adjust Trigger Level knob to stabilize displayed signal.
- Use cursors to determine time constant of Vc during charging, may need to change horizontal sensitivity for accuracy, and record in Table 5-1.
- Capture scope display of Vc using Run IntuiLink Data Capture desktop icon (ask instructor for steps after double-clicking icon).
- Observe the effect on the charge curve as R is changed to 30KΩ and then changed to 50KΩ.
  - Capture scope displays for both above R values to help answer a post-lab question.
- Set R back to 40KΩ and change square wave to a minimum value of 0 V and a maximum value of 5 V.
  - Observe effect this has on charge curve and capture to help answer a post-lab question.
- Change Channel 1 trigger setting to falling edge and determine time constant of Vc during discharging and record in Table 5-1.
- Capture scope display of Vc.
• Observe the effect on the discharge curve as R is changed to 30KΩ and then changed to 50KΩ.
  o Capture scope displays for both R values to answer a post-lab question.
• Set R back to 40KΩ and change square wave to a minimum value of 0 V and a maximum value of 5 V.
  o Observe effect on discharge curve and capture to answer a post-lab question.
• Calculate percent error between theoretical (done in prelab #2 & #3) and experimental charging and discharging time constants. Record in Table 5-1.

![Fig. 5-5 RC Transient Circuit](image)

<table>
<thead>
<tr>
<th>Measured</th>
<th>R(40KΩ) (Ω)</th>
<th>C(0.02µF) (µF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charging (µs)</td>
<td>Discharging (µs)</td>
<td></td>
</tr>
<tr>
<td>Measured Time Constant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculated Time Constant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percent Error (%)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5-1 R & C Measured Values / RC Transient Time Constant Measurements & Calculations

**Procedure 2: Capacitors & Inductors at DC Steady-state**

• Measure all resistors (Ω) used to construct the circuit of Fig. 5-6 and record in Table 5-2a.
• Calculate and record $R_{EQ}$ at nodes a-b using ideal DC equivalents for inductors (short) and capacitors (open).
• Construct the circuit of Fig. 5-6, except for the voltage source (power supply).
• Measure the open circuit resistance across terminals a-b. This yields the equivalent resistance $R_{EQ}$.

![Fig. 5-6 Capacitors & Inductors at DC steady-state Circuit](image)
### Table 5-2a DC Inductor and Capacitor Circuit: Resistance Measurements

<table>
<thead>
<tr>
<th>Resistances</th>
<th>( R_1 ) (25( \Omega ))</th>
<th>( R_2 ) (10( \Omega ))</th>
<th>( R_3 ) (130( \Omega ))</th>
<th>( R_{EQ} = R_{ab} ) (( \Omega ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculated</td>
<td>( R_{ab} = )</td>
<td>Percent Error ( R_{ab} = )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Calculate the voltage across each resistor using ideal DC equivalents for inductors (short) and capacitors (open). Record calculated voltages in Table 5-2b.
- Connect the 10V power supply (0.5A current limit) and measure the voltages across each resistor. Record measured voltages in Table 5-2b.
- Calculate percent error between measured and calculated voltages and record in Table 5-2b.

### Table 5-2b DC Inductor and Capacitor Circuit: Voltage Measurements and Calculations

<table>
<thead>
<tr>
<th></th>
<th>( R_1 ) (25( \Omega ))</th>
<th>( R_2 ) (10( \Omega ))</th>
<th>( R_3 ) (130( \Omega ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured Voltage (V)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculated Voltage (V)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percent Error (%)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Discussion

1. In Procedure 1, for all practical purposes, how long (in ms) does it take for the capacitor to fully discharge? Use measured values.
2. Describe the effect changing R in procedure 1 had on charging and discharging curves.
3. For procedure 1, describe the effect changing square wave amplitude from 0V to 5V had on charging and discharging curves.
4. In Procedure 1 when \( R = 40K\Omega \) and the square wave has minimum value 0V and maximum value 5V, if the frequency of the square wave is increased to 200 Hz, will the capacitor charge fully to 5V? Why or why not?
5. In Procedure 2, what is the primary reason for differences between measured and calculated values of resistor voltages? Hint: non-ideal storage element.