1. Determine $V$ and $I$ for the circuit shown below. Also, determine $V$ and $I$ when a short is connected between terminals a-b. All resistances are in ohms $\Omega$.

2. For the following circuits, determine $R_{TH}$ by finding the equivalent resistance “seen” by the load at terminals a-b. All resistances are in ohms $\Omega$.

Circuit a

Circuit b

Circuit c
3. Determine the Thevenin equivalent circuit (both $V_{TH}$ and $R_{TH}$) at terminals a-b. For the Thevenin resistance $R_{TH}$, find the equivalent resistance “seen” by the load. All resistances are in ohms $\Omega$.

![Diagram of electrical circuit]

4. For the circuit of Problem 3, determine the Norton equivalent circuit (both $I_N$ and $R_N$) at terminals a-b by connecting a short between terminals a-b and solving for $I_{SC} = I_N$.

5. The Thevenin resistance $R_{TH}$ can also be calculated using Ohm’s law equation below. Use the calculated values of $V_{TH}$ and $I_{SC}$ from Problems 3 and 4 to determine $R_{TH}$. Does $R_{TH}$ equal the $R_{EQ}$ value “seen” by the load?

$$R_{TH} = \frac{V_{TH}}{I_{SC}}$$
6. Given the following circuit: All resistances are in ohms $\Omega$.
   a. What $R_L$ value will provide maximum power transfer to the load?
   b. Calculate the power in Watts delivered to the load for the value of $R_L$ of part a.
   c. It is desired to provide maximum power transfer to a 10$\Omega$ load resistance ($R_L$). If a resistor ($R_4$) is placed in parallel with $R_2$, what is the required $R_4$ value?
**Objective**

To demonstrate that complex circuits can be simplified using Thevenin and Norton equivalent circuit techniques. To experimentally confirm Maximum Power Transfer Theorem applied to resistive circuits (i.e., maximum power delivered to load when $R_L = R_{TH}$).

**Workbench Equipment**

- DC Power Supply, Agilent E3640A
- Digital Multimeter, Agilent 34401A
- Resistor Box II, 10Ω/25Ω/40Ω/130Ω/269Ω/562Ω
- Resistor Decade Box, 1Ω and 10Ω step

**Check-out Equipment, 20-111 window**

- Banana to banana, 3 pairs, red/black
- Short leads, quantity 6, 1 bag

**Background**

For many relatively complex electrical circuits, it is often necessary to determine the current through and voltage across a load resistor as a function of load resistance. To simplify these calculations, it is useful to reduce the original circuit to either a Thevenin or Norton equivalent circuit.

**Thevenin Equivalent**

For many electrical circuits, only the external terminals are accessible. Internal circuitry is sealed inside a “black box,” see Fig. 4-1a.

According to Thevenin’s Theorem, it is possible to replace this box with a voltage source $V_{TH}$ and a series resistance $R_{TH}$ between terminals a-b. Thevenin voltage $V_{TH}$ can be determined by measuring or calculating the open-circuit voltage $V_{oc}$ across terminals a-b. $V_{oc}$ is the voltage across terminals a-b with $R_L$ removed creating an open-circuit. Short-circuit current $I_{SC}$ is determined by measuring or calculating the current through a short-circuit connected across terminals a-b. Short-circuit current is by definition equal to the Norton current $I_N$. In addition, $R_{TH}$ and $R_N$ are identical.

Thevenin resistance $R_{TH}$ (and therefore $R_N$) can be calculated using Ohm’s Law:
For the circuit of Fig. 4-1b, when load resistance $R_L$ is connected across terminals a-b:

\[
V_{TH} = IR_{TH} + IR_L
\]  \hfill (4-2)

\[
I = \frac{V_{TH}}{R_{TH} + R_L}
\]  \hfill (4-3)

where $I$ is the current through both the Thevenin and load resistors. For transparent black boxes (internal circuitry is visible), circuit simplification methods may be used to determine $V_{TH}$ and $R_{TH}$. Simplification methods, such as, KVL, KCL, $R_{EQ}$ - equivalent resistance “seen” by the load, Ohm’s Law and superposition method.

**Norton Equivalent**

Norton’s equivalent is another method used to simplify complex circuits. Fig. 4-1c shows a circuit equivalent to Fig. 4-1a represented by Norton current source $I_N$ in parallel with Norton resistance $R_N$. The Norton current can be determined by Equation 4-1 above.

**Maximum Power Transfer**

Maximum power transfer is an important concept in the development of many electrical circuits including home stereo speakers and computer interfaces. The concept is used to maximize power transfer to the load in these circuits. Thevenin or Norton techniques are applied to complex circuits to identify the optimum load resistance value ($R_L = R_{TH}$). The power transferred from the source to the load in a Thevenin equivalent circuit (Fig. 4-2) is given by:

\[
P_L = I^2R_L = V_{TH}^2 \frac{R_L}{(R_{TH} + R_L)^2}
\]  \hfill (4-4)

To find the value of $R_L$ that yields a maximum $P_L$:

\[
\frac{dP_L}{dR_L} = 0
\]  \hfill (4-5)

\[
\frac{dP_L}{dR_L} = V_{TH}^2 \frac{(R_{TH} + R_L)^2 - 2R_L(R_{TH} + R_L)}{(R_{TH} + R_L)^3} = V_{TH}^2 \frac{R_L^2 - R_{TH}^2}{(R_{TH} + R_L)^3}
\]  \hfill (4-6)

Thus, maximum power transfer occurs when $R_L = R_{TH}$. 

![Fig. 4-2 Thevenin Equivalent Circuit](image)
Procedure 1: Thevenin Equivalent

- Measure all resistors used to construct the circuit of Fig. 4-3 and record in Table 4-1a.
- Construct the circuit of Fig. 4-3.
- Disconnect source leads at source terminals and then connect source leads together (“eliminates” source). Measure the open circuit resistance across terminals a-b.
  - This yields the Thevenin equivalent resistance $R_{TH}$. Record in Table 4-1a.
- Calculate the expected $R_{TH}$ value and record in Table 4-1a. Also calculate percent error.

![Fig. 4-3 Circuit Used to Determine Thevenin Equivalent](image)

<table>
<thead>
<tr>
<th>Resistances</th>
<th>$R_1$ (130Ω)</th>
<th>$R_2$ (40Ω)</th>
<th>$R_3$ (25Ω)</th>
<th>$R_4$ (10Ω)</th>
<th>$R_{TH} = R_{ab}$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured (Ω)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculated $R_{TH}$ (Ω)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Percent Error $R_{TH}$ =</td>
</tr>
</tbody>
</table>

Table 4-1a Thevenin Equivalent Resistance Measurements

- Set the Agilent E3640A power supply to 10V with a 0.5A current limit.
- Connect the power supply to the circuit and measure the open-circuit voltage across the a-b terminals. Record the result in Table 4-1b.
- Calculate expected $V_{TH}$ and enter in Table 4-1b. Also calculate percent error.

<table>
<thead>
<tr>
<th>Measured $V_{TH}$ (V)</th>
<th>Calculated $V_{TH}$ (V)</th>
<th>Percent Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4-1b Thevenin Voltage Measurements and Calculations

Procedure 2: Norton Equivalent

- Measure the short-circuit current at the a-b terminals by replacing $R_L$ with an ammeter.
  (Agilent 34401A set to measure DC current). Recall ideal internal ammeter resistance = 0Ω.
  - Note: ammeter inputs red I and black LO must be used (located lower far right).
- Calculate expected $I_{SC}$ and enter in Table 4-2, along with percent error.

<table>
<thead>
<tr>
<th>Measured $I_N$ (A)</th>
<th>Calculated $I_N$ (A)</th>
<th>Percent Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4-2 Norton Current Measurements and Calculations
Procedure 3: Thevenin Equivalent Circuit Measurements

- Using measured $V_{TH}$ and $R_{TH}$ values from Procedure 1, construct the circuit of Fig. 4-1b.
  - Use the circuit of Fig. 4-3 for $R_{TH}$. Replace 10V source with a short. **Disconnect source, do not short power supply!** Resistance between nodes a and b = $R_{TH}$.
  - Make certain to adjust source voltage to $V_{TH}$, do not leave at 10V.
- Adjust $R_L$ (1Ω step decade box) as close as possible to $R_{TH}/2$ and record both nominal (desired) and measured (actual) resistances in Table 4-3.
- Connect the resistance to the a-b terminals. Measure the voltage $V_{ab}$ and record in Table 4-3.
- Repeat above two steps for load resistance values of $R_{TH}$ and $2R_{TH}$.
- Calculate the expected values for $V_{ab}$ for the three $R_L$ values and record in Table 4-3 along with percent error.

<table>
<thead>
<tr>
<th>Nominal $R_L$ (Ω)</th>
<th>Measured $R_L$ (Ω)</th>
<th>Measured $V_{ab}$ (V)</th>
<th>Calculated $V_{ab}$ (V)</th>
<th>Percent Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{TH}/2$ =</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_{TH}$ =</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2R_{TH}$ =</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 4-3** Thevenin Equivalent Circuit Measurements and Calculations

Procedure 4: Maximum Power Transfer

- Continue to use the circuit setup of procedure 3.
- Fill in Table 4-4 with Nominal $R_L$, Measured $R_L$ and Measured $V_{ab}$ for $R_{TH}/2$, $R_{TH}$ and $2R_{TH}$ from Table 4-3.
- Measure the voltage across the a-b terminals for eight additional $R_L$ values.
  - Make four of these additional measurements between $R_{TH}/2$ and $R_{TH}$ with one measurement an ohm less than $R_L = R_{TH}$ and the other three measurements approximately evenly spaced between $R_{TH}/2$ and $R_{TH} - 1Ω$.
  - Make four of these additional measurements between $R_{TH}/2$ and $2R_{TH}$ with one measurement an ohm more than $R_L = R_{TH}$ and the other three measurements approximately evenly spaced between $R_{TH} + 1Ω$ and $2R_{TH}$.
- For each additional $R_L$ value, record the nominal and measured $R_L$ values, and measured $V_{ab}$ in Table 4-4.
- Use the measured values of both $R_L$ and $V_{ab}$ to calculate the power delivered to the load
  $$P_L = \frac{V_{ab}^2}{R_L}.$$ Record all values in Table 4-4.
- Using Excel, plot the load power $P_L$ as a function of $R_L / R_{TH}$. 


### Procedure 5: Power Supply Internal Resistance

- Set the Agilent E3640A power supply to 0.5V with a 0.5A current limit.
- Measure the resistance of two leads to be used to connect the power supply to a decade box. Record in Table 4-5.
- Measure the resistance of a 1Ω step decade box set at 1Ω, 2Ω and 3Ω; record in Table 4-5.
- Connect decade box to the power supply terminals.
- With decade box set to 1Ω, record power supply current. Note: Record the power supply current display value, do not measure with ammeter.
- Measure and record the voltage across the decade box. Record as $V_{RL}$ in Table 4-5.
- Repeat previous two steps for decade resistance settings of 2Ω and 3Ω.
- Determine the power supply’s internal resistance for each measurement. Use the measured $V_{RL}$ in your calculation and be sure to account for cable resistance. Record all calculated values in Table 4-5.

<table>
<thead>
<tr>
<th>Nominal $R_L$ ($\Omega$)</th>
<th>Measured $R_L$ ($\Omega$)</th>
<th>Measured $V_{RL}$ (V)</th>
<th>Measured $I_S$ (A)</th>
<th>Calculated $R_{int}$ ($\Omega$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{TH}/2$ =</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_{TH}$ =</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2R_{TH}$ =</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

**Table 4-4 Load Power as a Function of Load Resistance**

<table>
<thead>
<tr>
<th>Nominal $R_L$ (Ω)</th>
<th>Measured $R_L$ (Ω)</th>
<th>Measured $V_{RL}$ (V)</th>
<th>Measured $I_S$ (A)</th>
<th>Calculated $R_{int}$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$R_{lead} = \Omega \quad R_{lead} = \Omega$

**Table 4-5 Power Supply Internal Resistance Measurements and Calculations**
Discussion

1. Determine the equivalent resistance “seen” by the voltage source of Fig. 4-3 with \( R_L \) disconnected (use nominal R values). Is this resistance the same as the resistance “seen” by the load with \( R_L \) disconnected? If not, why different?

2. Use measured \( I_N \) from Table 4-2 and measured \( R_L = R_{TH} \) value from Table 4-4 and calculate maximum power transfer to the load using a Norton equivalent circuit analysis. How does answer above compare to corresponding \( P_L \) in Table 4-4?

3. What is the main advantage of Thevenin and Norton equivalent circuits?

4. Comment on the \( P_L \) vs. \( R_L / R_{TH} \) plot of Procedure 4. Is the curve as expected? Why or why not?

5. Compare cable resistance to the power supply’s internal resistance. Calculate percent error introduced by the cable resistance. In other words, calculate the percent error introduced if the cable resistances were not accounted for (i.e., the cables were considered to be ideal).

6. In procedure 5, why is it necessary to measure voltage across the decade box? Why not use the power supply display voltage?